Order and Structure of Islamic Pattern

Loai Morshed Dabbour، Department of Architecture, Faculty of Architecture and Design, AlZaytoonah University of Jordan, Jordan

تاريخ الاستلام:8 /2020/6 تاريخ القبول: 2020/12/1

نظام وبنيوية الزخرفة الهندسية الإسلامية

لؤي مرشد دبور، قسم هندسة العمارة، كلية العمارة والتصميم، جامعة الزيتونة الاردنية، الأردن

الملخص

تهدف هذه الدراسة إلى فهم نظم إنشاء الزخرفة الهندسية الإسلامية ودراسة أنماطها الجيومترية من خلال دراسة تراكيب شبكاتها من تقاطعات الخطوط والدوائر وأثرها في نظام وبنيوية إنشاء الزخرفة، ولمناقشة القوانين والنظم التي شكلت البنية الجيومترية للمنظومة الزخرفية قي تراكيب النسب الفاضلة ومعانيها. وهذا يشير إلى الحاجة إلى دراسة المبادئ والقوانين المولدة لتشكيلاتها وتعدد أنماطها. ولفهم هذه القواعد تم دراسة عدد من الحالات الزخرفية لاشتقاق أنماطها المتعددة بناء على مفهومي النظام والبنيوية ودورهما في إنشاء معماريةِ وجيومتريةِ العناصر الزخرفية وأنماطها. فقد أشارت تحاليل هذه الزخارف إلى أن تعدد التراكيب والأنماط يرتبط بنظم وقواعد الشكل الأساسي من حيث تضاؤلاته وتراكيبه وتقسيماته. وبذلك يقدم البحث مقاربة لدراسة الزخرفة الإسلامية بفهم أعمق لآلية بناء تراكيب التشكيلات الزخرفية، وهذا يؤدي إلى امتلاك المصمم للقدرة على تحديد مسارات الأشكال في بناء النمط الزخرفي. وبذلك فإن شبكات الخطوط المولدة للأشكال مرتبطة بمفهوم النظام كمحدد لتراكيب الشبكات المكونة للشكل ولمفهوم البنيوية لتشكيل قراءة أوضح لتكوين الأنماط الزخرفية الهندسية. هذا البحث أظهر أن التصاميم التقليدية باستخدام أشكال مضلعة بسيطة ومتوالياتها أدت إلى تحديد قوانين ومبادىء سهلت عملية تصميم التشكيلات الزخرفية المعقدة الناشئة منها.

كلمات مفتاحية: النظام، البنيوية، الجيومتري، النسب الهندسية، الزخرفة الهندسية.

Abstract

The purpose of this research is to discuss the evolution of the Islamic pattern geometry in terms of the grid layout and how it affects the order, structure, and eventually the construction of the pattern. This fulfills the need to study the generation forces of Islamic patterns and the laws behind its compositions and tessellations. The proposed pattern analysis suggests that the pattern variations are based on polygon rules and its diminished overlaps and subdivisions, that allows the pattern variation formations. The research presents an approach to study the Islamic pattern symmetries, that provide a deeper understanding of the structure of pattern variation, allowing designers to achieve improved control over the pattern composition and structure and argues that in pattern design the grid layout is strongly linked to order as spatial grid layout within the geometric proportional configuration and to structure as an intelligible configuration in the construction of the pattern design tessellations. This paper shows that traditional designers, using simple consecutive polygon geometry, have resolved the complicated principles of pattern formations.

Keywords: order, structure, geometry, proportions, Islamic pattern.

Doi : https://doi.org/10.47016/14.3.6

Introduction:

Researchers argue that pattern geometry manifests mathematical ratios and proportions which is central to Islamic art. In this argument Islamic pattern is a manifestation of the harmonic subdivisions of the circle and its template grids (El-Said, 2001, Critchlow,1976, Akkach,2005).

This paper studies the concepts of order and structure as systems employed in the construction of Islamic pattern design. Our purpose is to describe the architecture of Islamic pattern, and to explore the evolution of the pattern geometry in terms of the grid layout of intersecting circles and lines propagation and linkages that affect the order, structure, and eventually the architecture of the pattern.

Among various systems developed for the study of Islamic patterns, order and structure concepts are highlighted due to their efficiency in establishing the pattern construction possibilities. Order and structure in Islamic pattern are concerned with the different ways motifs can fit together to form the pattern's architecture. By comparing the variations on the pattern-construction we shall aim to reveal the extent to which patterns adopted the same order with a consequence of mathematical harmony of the traveling lines and node intersections. This calls for knowledge about order and structure of pattern variations behavior and its dependence on topological characteristics of pattern designs.

Order is the expression of the discipline of geometry and is represented in the parameters and laws of the grid tissue. Structure is both quantitative and qualitative; its quantitative dimension regulates the order which is the base of pattern design construction. Its qualitative aspect regulates the proportions as the aesthetic value of design form an expression of the pattern architecture (Dabbour, 2012). Order represented by the geometric grid system is the blueprint and the generator of all pattern forms. Grid layout explores and enlightens the pattern's construction that unifies and reveals the natural growth of the pattern construction through conscious selection (contemplation) of the grid intersections and its conformation to geometric shapes. All pattern forms emerge out of sensible geometric grid codes. Understanding and contemplating these codes allow us to comprehend the wisdom of the inner working systems of the pattern construction to fully understand and appreciate the beauty of the different pattern construction configurations (Critchlow, 1976, 1987, Broug, 2019).

Structure in pattern architecture represents a configuration language as letters, words and sentences do in a spoken language. The use of language is bounded by the grammar that allows for intelligibility. Structure is the pattern grammar that determines the relationships of intersecting lines and circles (order) to regulate the contexts within which elements and shapes may be arranged in a pattern. The relationships of the elements to each other, and the relationships within all elements addresses and reflects the natural laws that direct the basic design of the pattern configurations. This language springs mainly from the abstract and is based on geometry and geometrical proportion ruled by unseen laws (proportion) (El-Said, 2001, Prange, 2009) (fig.1).

In Islamic pattern, laws of order represent the surface topography modeled by a basic grid of lines and circles subdivided into modules according to the layout of the basic grid construction. Each module is demarcated by a single level of order that extends over multiple repetitions that will accurately estimate the proportions provided by the grid. They are governed by the proportional analysis systems which are based on length and width configurations. This is a target proportional criterion most efficiently based on proportional design methodology for pattern systems (Dabbour, 2012, Broug, 2016). As pattern form becomes more intrinsic, finding an appropriate system (structure) for better performance and constructability is essential to carry out the pattern design variations. The grid order arrangement has great possibilities to be developed as one of the most appropriate solutions for pattern construction and, at the same time, it is a very

challenging task to accurately define and construct any variations in the pattern process due to its complex geometry.

Circle: (Heaven / Spirit)
Completeness and Perfection Motion

Triangle: (Man / Soul)
Harmony Space

Square: (Earth) Body
Stability Time

Fig. (1): Pattern language Architectural vocabulary

Geometry and Geometric Proportional Systems

Geometry is a hidden manifestation in Islamic pattern. It is the visual means by which we understand the order and harmony in pattern architecture. It is a representation of the mathematical orders (universal truth) with their aesthetic and metaphysical values, and applicable to all aspects of Islamic pattern design procedure (Schuon, 2003, Lawlor, 2002). The construction of a pattern is established and displayed by defined mathematical and geometrical constants based on the essential harmonies of perfect proportions that are underlying the fabric of the pattern design. By contemplating and understanding these geometric proportions we could obtain visual codes to the construction formations of a pattern process. This is noted in the discovered Topkapi Scrolls (Timurid manuscript dating from the 15th century), which shows that early Muslim craftsmen developed theoretical rules for the use of aesthetic geometry (Necipoglu, 1995). Ikhwan Al-Safa argued, "One of our aims consists of demonstrating clearly that the whole world is composed in conformity with arithmetical, geometrical, and musical relations which form the reality of universal harmony" (Ardalan, 2000). Geometric ratios and proportions are the main principles employed in the design and construction of Islamic pattern with parameters that integrate mathematical figures, constants and ratios such as the proportional rectangles (roots), golden (sacred) mean, Pythagorean triangle, and the primary three proportional roots √2, √3 and √5. All Islamic geometric pattern designs are based on these proportions (Critchlow, 1987, Broug, 2013). The golden mean and the proportional rectangles or the proportional roots are based on the geometry of the polygons. The square includes root √2, the Hexagon includes root √3, and the Pentagon includes the golden mean, which is quantitatively equal to [(root √5+1)/2] = 1.61803 (fig. 2) (El-Said, 2001, Critchlow, 1976, Dabbour, 2012). In Islamic pattern, geometric proportions have been employed and used to create unique pattern formations and became the main design tool based on the parameters of the concepts of order and structure. All these geometric proportions can be constructed within a circle (circle of unity) which includes all main roots and proportions. Figure (3) shows the relationship represented by two overlapping circles. The first circle represents the absolute (the metaphysical) and the second circle represents the relative (the illusion). The intersection of these two circles is known as the Vesica Pisces and according to Schuon, is the "Relatively Absolute" (Schuon, 2003). The geometric proportions evolved in the "Relatively Absolute" construction represent the proportions of sacred geometry that were applied in the geometric matrix to create orderly principles used for the design of all Islamic patterns. This mathematical harmony has a cosmological manifestation as can be

seen in the universe via the proportions of the planets in our solar system. For example, the ratio of root 3 (□3) proportion can be seen in the rotations of Jupiter and Earth orbits around the Sun in that, as shown in figure 3, the result of this motion constructs rectangles within this configuration based on root 3 proportion (Martineau, 2001).

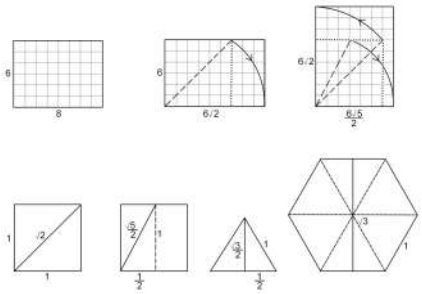

Fig. (2): Pythagorean triangle, and the primary proportional roots shown on a grid system

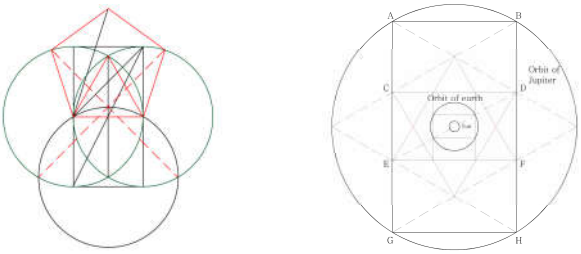

Fig. (3): The roots and the golden mean within the Vesica, and The rotations of Venus and Earth around the Sun proportions

Order and structure: Pattern morphology

The generation of pattern construction is based on exploring a variety of configurations based on the defined laws (order) embedded in the grid intersection positions. Examples are illustrated in Fig. 4. The different tessellations of 4-8-16-32-fold geometric star patterns can be used directly to explore different constructions in the case of different line/node intersections selections. This is a contemplation journey within a basic grid integration.

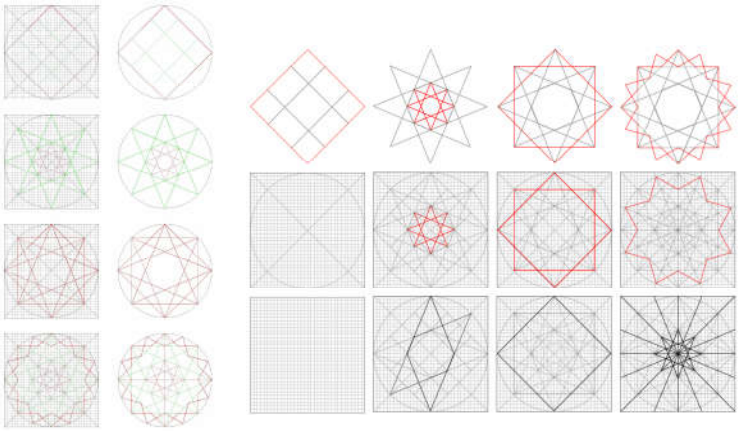

Fig. (4): Exploring different pattern tessellations based on four-fold grid layout configurations

However, for methodical reasons the pattern grid module is related to certain constraints; First, the allocation and the exact position of springing pattern elements based on intersections between different lines in the polygon grid patterns. Second, the

allocation of nodes margins from the proper interpretation of line intersections in the pattern systems. In practice, the resulting quality and quantity of line intersections seem to produce a variety of patterns that would allow picking out a multifaceted suite of pattern construction (order). Thus, the identified intersection nodes actually represent different pattern construction variations (structure). Models based on the module gridline intersections can be used to define the pattern construction variations. Alhough the construction of Islamic geometric patterns is based on constructive polygons, (most common ones are the pentagon, hexagon, and octagon), the polygons star configurations are formed and constructed by connecting the vertices of intersections and nodes of the diminishing of the polygons intersecting lines. Hence patterns are differentiated and labeled according to the ordinated polygon such as 4-8-12, 5-10, 6-12-pointed star geometrical pattern (fig. 5) (Broug, 2016, El-Said, 2001, Abdullahi, 2013). The construction of Islamic patterns in its simple shapes follows a direct order principle from polygons grid parameters path construction, but the complex constructions are based on more complicated intersecting nodes generated from the diminishing of the polygons grid orders. The relationships between grid intersections and spatial nodes of the pattern are shown in Fig. 5. In conjunction with their spatial distribution, these relationships could be helpful to identify structural elements within pattern transects. However, geometric properties of a set of intersecting nodes account for all pattern construction variables. The significance of pattern configuration evokes specific sets of nodes properties for each variable. The hierarchies of ordering intersecting lines and nodes define the levels of pattern construction and its methodical derivation applying a correspondingly orderly pattern design procedure. The establishment pattern geometry as a validation tool involves multiple steps which are: first, characterization of the prototyping matrix; second, construction of prototype replicas; third, development of the pattern material properties; and fourth, comparison of results from replicas at the apparent pattern structure level (Broug, 2019).

Pattern structure is more often governed by order requirements in relation to the proportional analysis of a specific construction system for a pattern with a certain height to width ratio of design aesthetics, which is primarily an amalgamation of a particular length and width configuration. With this proportion of configuration, design can meet the target proportional criteria most efficiently. Based on proportions, a design process for a pattern construction system could be evolved. A pattern structure is modeled as a grid line/circle subdivided into modules according to the repetitive basic pattern. Each module is defined by a single level of order that extends over multiple repetitions. To more accurately estimate the proportions provided by the grid, Figure 6 illustrates the case of a ten-fold module, in which the size of the pattern is generally determined in the proportions between the width and the height parameters. The design begins by specifying the desired pattern structure based on the required proportions allocated to the perimeter grid and pattern structure. Consequently, patterns are designed to have more repeated basic shapes (Hambidge, 1920).

The illustrations of the ten-fold geometric construction configuration methodology, as can be noted from the design construction studies of the pattern constructability in figure 6 shows that as the rate of order repetition increases, the construction of the pattern becomes more challenging. Indeed, various aspects should be well-thought-out in an integrative way with multidimensional grid relationship to successfully carry out complex-pattern designs. The basic constraints involved in the construction of a pattern are: first, topology, second, internal intersection and intersecting node locations, third, external pattern frame parameters, fourth, configurational arrangements, and fifth, geometrical shape constraints. Amongst these, topology defines the connectivity of grid

intersections and nodes as the geometrical configuration is described in terms of intersecting nodes configurations. Determination of geometric construction configuration at the structure level is known as form-finding or shape construction composition, which is considered a key step in the design of pattern systems. Geometrical symmetry properties are often needed to ensure a unique solution of the pattern form construction. The set of pattern forces can be specified due to the pattern construction equilibrium conditions and geometric constraints found by investigating the connections of the line intersection matrix.

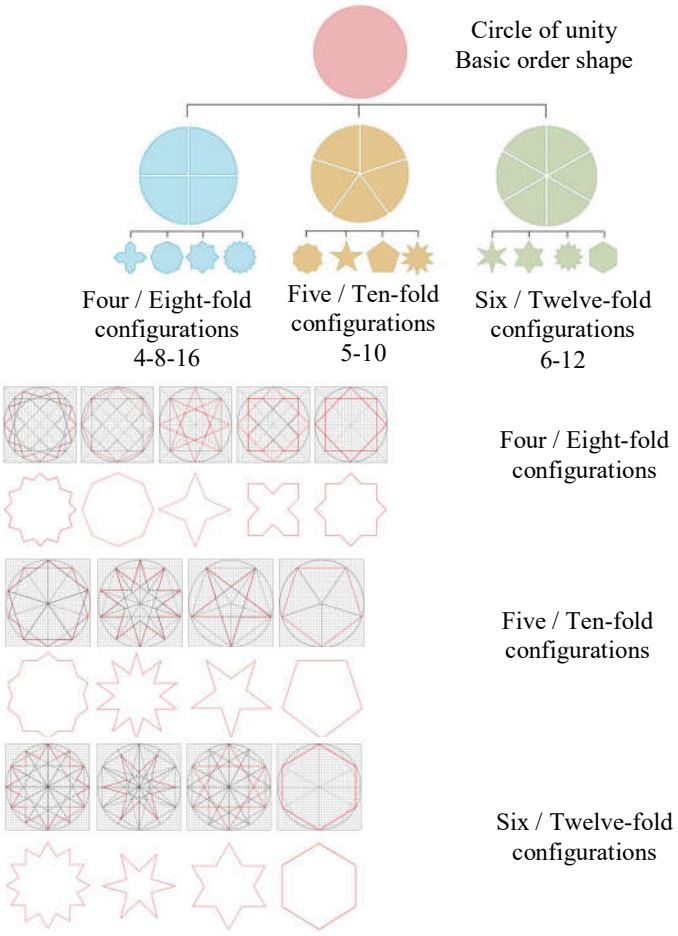

Fig. (5:) The relationships between grid elements and spatial extent of the pattern configurations

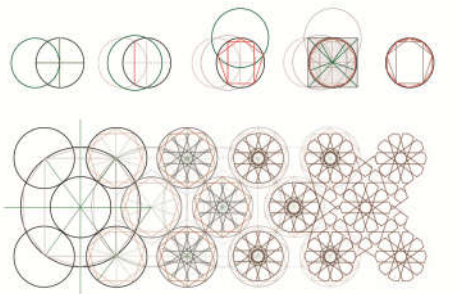

Fig. (6): Ten-fold pattern repetitions modules

In designing a pattern system, the dimension/size of the pattern should be specified in order to present a unified approach to shape the design of the pattern construction. The locations of the intersecting grid nodes are to be fixed to transform the intersecting lines into a pattern system. The design procedure could be sequenced as follows: First, generate the grid system by establishing the circle of unity with the basic polygon grid structure. Second, specify the topology. Third, assign the directions of intersecting grid lines, and circles so as to define the pattern layouts. Give the symmetry properties by the intersecting grid matrix. Fourth, specify and obtain the pattern by using/selecting the shape configurations. Fifth, remove the working lines and the pattern will manifest (fig. 6).

Order and Structure: Study Model

The concepts of order and structure as a tool of the Islamic pattern design process are based on order as spatial grid layout and structure as an intelligible pattern construction configurations and aesthetic criteria. These two notions are integrated into the conceptual phase of the pattern design process. This serves two objectives; firstly, in pattern construction, grid layout is strongly connected to geometric order as a self-guiding method of design. Secondly, structure is the intelligibility of the synthesis of the pattern configurations that regulates the order of the pattern, and at the same time, it mediates between order and diversity at the aesthetic level. Order which is inherent within a spatial arrangement and the structure of the arrangement in Islamic pattern visualizes the relationship between the grid intersections that comprise it and determines the construction of every pattern. This means going back to the basic concepts, to the origins, to search for the laws behind it, and to try to isolate the principles underlying its formal organization to get behind the overall impression to the conscious order that governs and informs the whole. Order in the general overall scheme of things does not seem to guarantee intelligibility of the pattern architecture; rather, it is the opposite. Once the observer is located within the spatial configuration and therefore is able to experience the pattern space as a series of related spatial discontinuities, geometrical order can confuse and disorientate. The grid layout generally is ruled by a simple repetitive order formula which enforces symmetry on the design composition. Such a grid layout may be square, circle, pentagonal, or hexagonal, embodying various design variations as well as complex spatial constructions. The pattern form which springs out from a maze-like gridline intersection can be traced through the medium of the grid basic order, which reveals the deliberate intentions or conscious inspirations of the designer (fig. 7).

It is argued here that what makes the pattern configuration intelligible to the user is the structure of the pattern layout and not its visual order at all. Structure, it is suggested, is different from order. The view from the grid layout, to an observer within the pattern configuration experiencing it bit by bit, requires movement to make sense of the layout, and intelligent movement in turn requires the layout to provide information not just about where the viewer is locally located at line intersections and nodes composition, but also about where he might potentially be located within the overall scheme of the pattern's intersecting nodes.

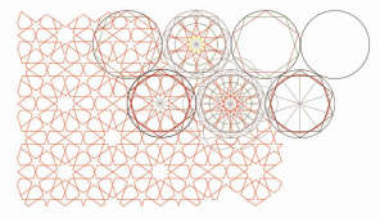

Fig. (7): The pattern springs out from a maze like grid line intersections

This distinction between order and structure is made clear by the following example. Figure 8 illustrates a four-eight-fold square-framed grid layout development which begin with a simple grid geometric shape constructed from lines and tangent circles framing the basic square pattern shape (fig. 8). This is a representation of the basic method of polygons, stars, and pattern construction dated to the 12th century based on a simple one layer of the ordered grid (Necipoglu, 1995). This grid then evolves by the introduction of multi-layer grid systems based on the diminishing of the polygon grid lines. By that, a more dynamic and complex hybrid grid layout is developed. Within this maze of intersecting nodes, the structure of the configured pattern is developed by isolating and associating the sub-grid formations to the original grid central nodes, but under a different scale. This process requires a knowledge of the theoretical rules that govern the design and construction of esthetic geometry (structure). These rules are based on the dimensions (length and width) of the design panel, the material of the final design application, and the scale of configured pattern geometric pieces on the one hand, and, on the other hand, the final panel proportional systems and symbolic meaning.

A more detailed pattern (8-fold geometrical pattern variations) is developed through multipoles of primary 4-8-point star patterns. This varies from simple patterns to highly complex pattern compositions that are evolving from different layers of grid compositions. Figure 8 shows the tessellations of 8-fold pattern variations that are developed on an underlying multi-sub-grid foundation (order). The resulting variations are dependent on the topology of the grid patterns and the allocation of different intersecting nodes. Each group of intersecting nodes will direct the pattern into a new form as illustrated in (fig. 8). Four-eight-fold star pattern shapes are emerging from a combination of underlying grid patterns from a simple form to a highly complex form and this is a production of pattern structure; an experienced knowledge of geometry that can comprehend all the grid layers and surf for joint nodes that can connect to create different pattern variations based on the same order multi-grid construction. These illustrations are based on the basic square units to an eight-point geometrical pattern structured of interweaving pattern composition of polygons, based on a grid of diminished circles, and knit lines flowing in all directions. Each unit (node/center) is a self-regulating form process, in that the form is generated from the part to the whole within a coherent rhythm of the total pattern. This construction methodology requires considerable knowledge of practical geometry. In that, the pattern design is built upon a system of composite articulation that eventually can be reduced to a simple polygonal element. The pattern may be built up of a grid of arcs, radial lines, rectilinear lines combined together to work out simple or complicated patterns.

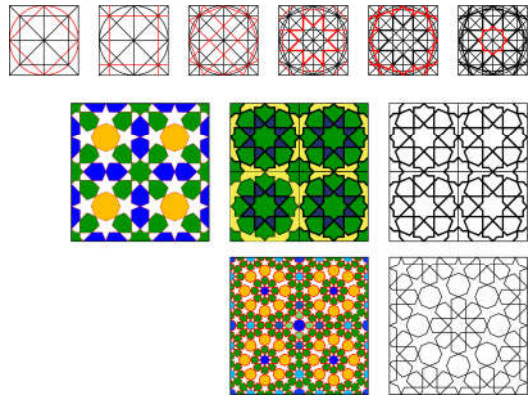

Fig. (8): four - eight-fold pattern order layout – a pattern language springs from the grid configuration

Figure 9 explores the construction of a ten-fold pattern layout. The construction procedure starts by establishing the pattern's size (length and width). From the pattern proportional outlines, a basic grid working lines are established via circles and intersecting lines that form the pattern layout. A set of these intersecting circles are then grouped around larger connecting nodes. Finally, a series of local connections are grouped around the central focal connection of the entire line-circle-node community. Every node and intersection seems to give identical information about its position within the overall network and, eventually, the designer has to learn the small visual differences which identify each local segment of the intersecting nodes structure and then to relate them together to the pattern layout. However, the layout of intersecting lines and nodes is not easily intelligible because all the local intersections and nodes are the same. Since orientation within the grid net is extremely difficult, the concepts of order and structure establish the pattern construction process. Order depends essentially on recognisable similarity of parts to yield an immediately available organized whole, whereas structure is the underlying pattern which is picked up by moving about the grid layout and which depends on an arrangement of differences. The relation between order and structure is differently constructed in each pattern design case. Order and structure are based on the differences within a configuration from one part to another, which makes intersecting nodes selection an intelligible process.

Structure is the pattern design language that evolves from a set of components and vocabulary that can define and combine coherently the construction of the pattern. This comprises the specific geometries that determine the shape and size of pieces that can be applied to particular materials. This is what characterizes the pattern construction style that evolves from a language that always adapts to the pattern application and uses and they form at the same time an essential part of cultural identity that conveys spiritual design aspects. Various design geometries can be freely invented within these constraints. This pattern language is based on proportional geometric relations revealed in evolved pattern typologies that associate geometric configuration with function and use. Thus, for a specific pattern design, a set of grid relationships has to be thought from the beginning (Steinhardt, 2007, Broug, 2019). The pattern's construction (structure) does not disclose the laws that make it evolve from hidden grid working lines. Evolutionarily the selection of intersecting nodes creates the morphological types of pattern environments that identify the archetype construction process for each different space and form.

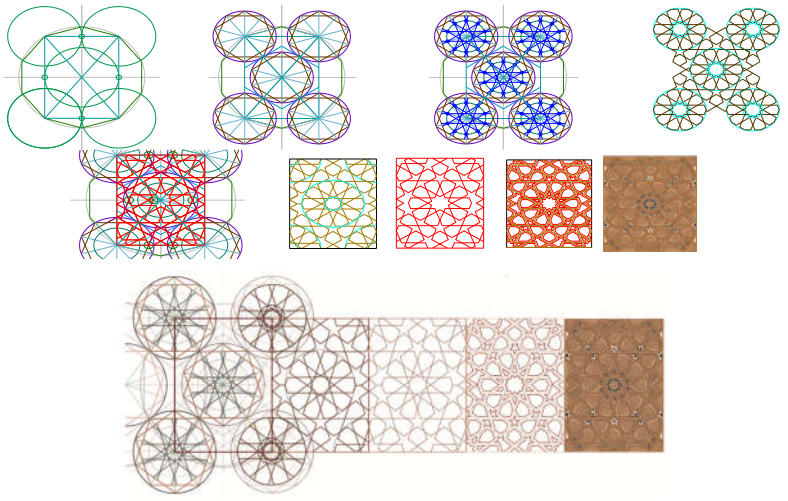

Fig. (9): Tenfold pattern order layout – a pattern language springs from the grid configuration

The pattern structure builds up organized complexity in an interactive cycle. This complexity in the pattern construction contains a comparable degree of the estimated number of connections (nodes) and the relations among these connections (nodes). This procedure is built-up in the grid order layers and, at the same time, it organizes the order to be intelligible with the pattern construction process. In that, pattern form relies upon the grid organized complexity for design innovation. As pattern evolves it builds an inherent degree of structured complexity required for the pattern functionality and use. In understanding this procedure, pattern construction may be navigated from abstract visual expression to a surprise that cannot be predicted as it springs from a contemplation process which is not a mechanical process. In this way, traditional pattern form evolved richness of expressions, to accommodate social and cultural interaction and higher applications of spiritual meaning. The traditional method of design begins by contemplating nature and cosmos that is felt by anyone who moves within the pattern structure. Islamic pattern can be created within this hypothesis: it is a contemplation journey of spiritual meaning, not a mechanical process, to influence how to generate a construction through which we could reveal concepts that involve the structure of the pattern as wholeness, not merely as a condition.

Discussion and Conclusion:

This paper presented the Islamic pattern construction model based on the concepts of order and structure. They have been used for construction, composition, and configuration of Islamic pattern design based on the construction characteristics of the polygon grid layout as order concept provides great potentials and variations to structural and aesthetic pattern context and compositions.

The interface of the designer during the grid generation process allows guiding the pattern geometry to evolve in the desired direction. The grid (order) will act as a direction towards the evolution of the grid multiple intersection nodes into a structural pattern that optimizes the selected performance criteria and value. Visualizing a proper selection of line-node intersections is a performance that leads the generation of the pattern design process. Design alternatives can be generated based on the selected range of variables and solutions in which the designer can surf, looking for a design direction (structure) and for aesthetic knowledge of the pattern performance criteria.

The study has shown that the pattern layout is based on a selection of line intersections on the grid system. Design alternatives are formed based on selecting variable intersecting grid lines and nodes. This is associated with grid performance in the revelation of different design alternative directions based on an assessment of different alternatives and understanding of the relationships between the geometric properties and the generation of alternate design solutions.

The evidence presented in this paper suggests that Islamic pattern construction was conceived and constructed based on the local polygon grid rules, in which pattern construction formations can grow by adding more intersecting nodes diminished from the polygon without changing the first-order set by the grid layout. And this can be used as a general guiding principle for constructing variable pattern variations that can be applied to different materials. In addition, this approach provides an easy tool for designers to generate and study complicated pattern formations based on the polygon symmetries construction lines.

Case studies have shown the generation and exploration of different pattern design alternatives from the base of basic order parameters. Pattern geometry initiated by the order layout enhances the knowledge of pattern structure design by permitting the designer to module variable pattern design aspects evolved at an early stage. This also allows the designer to establish consistent tangible thinking, which is crucial for the usability of the design process and application.

Pattern complexity in the grid order makes it essential for the designer to acquire knowledge and understanding of how to respond to alternate typologies to adapt to the organized complexity. Pattern language, just as spoken language is structured on grouping the right vocabulary in the right order, thus connecting the related group of intersecting nodes in order to liberate the pattern creativity and satisfy a variety of spiritual symbolism.

Future efforts should be directed toward investigating the Islamic pattern generations based on fixed panel dimensions and how these variable geometric models can be correlated with geometric proportional systems.

Sources and references **المصادر والمراجع**

1. Abdullahi, Yahya, Mohamed Rashid Bin Embi. (2013). *Evolution of Islamic geometric patterns. Frontiers of Architectural Research*, Vol. 2/2, str. 243–251.
2. Akkach, S. (2005). *Cosmology and Architecture in Premodern Islam: An Architectural Reading of Mystical Ideas*. State University of New York Press.
3. Ardalan, Nader, and Laleh Bakhtiar. (2000). *The Sense of Unity: the sufi tradition in Persian architecture*. Kazi publications.
4. Broug, Eric. (2019). *Islamic Geometric Patterns*. Thames & Hudson.
5. Broug, Eric. (2016). *Islamic Design workbook*. Thames and Hudson.
6. Broug, Eric, (2013). *Islamic Geometric Design*. Thames and Hudson.
7. Critchlow, Keith. (1987). *Order in space: a design source book*. Thames and Hudson, USA.
8. Critchlow, Keith. (1976). *Islamic patterns: An Analytical and Cosmological Approach*. Schocken Books, Thames and Hudson, London, UK.
9. Dabbour, Loai. M. (2012). *Geometric proportions: The underlying structure of design process for Islamic geometric patterns*. Frontiers of Architectural Research, Vol. 1/4, str. 380-391
10. El-Said, Isam. (2001). *Islamic Art and Architecture: The Systems of Geometric Design*. Garnet Publishing Ltd., UK.
11. Hambidge, Jay. (1920). *The elements of dynamic symmetry. Yale university press*.
12. Khawaja, Muhammad Saeed. (2011). *Islamic Art and its Spiritual Message*. International Journal of Humanities and Social Science, Vol. 1/2, str. 227-234
13. Lawlor, Robert. (2002). *Sacred Geometry: Philosophy and Practice (Art and Imagination)*. Thames and Hudson page 82.
14. Martineau, John. (2001). *A little book of coincidence*. Walker and company, New York, U.S.A.
15. Necipoglu, Gulru, et.al. (1995). *The Topkapi Scroll: Geometry and Ornament in Islamic Architecture*. Geny Research Institute
16. Prange, Sebastian R. (2009). *The Tiles of Infinity*. Saudi Aramco World, str. 24-31
17. Steinhardt, Paul, et.al. (2007). *Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture*. Science, Vol. 315/5815, str. 1106-1110
18. Schoun, Frithjof. (2003). *Survey of Metaphysics and Esoterism*. World wisdom, Indiana, USA.